Development of Web-Based Learning Media for Matrix Operations and 4×4 Determinant Calculation with an Adaptation of the Sarrus Method

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Abstrak

Tujuan dari penelitian ini adalah untuk mengembangkan media pembelajaran berbasis web yang dapat digunakan untuk menghitung operasi matriks (penjumlahan, pengurangan, dan perkalian) serta menghitung determinan matriks ordo 4×4. Sebagai alternatif dari metode kofaktor yang kompleks untuk perhitungan determinan, maka digunakan metode Sarrus yang telah diadaptasi. Penelitian dan pengembangan (R&D) menggunakan pendekatan pengembangan media pembelajaran digital dengan tahapan analisis kebutuhan, desain prototype, implementasi, dan evaluasi. Media pembelajaran ini mengintegrasikan operasi dasar matriks dengan pendekatan Sarrus yang diperluas melalui bagian A1, A2, dan A3 untuk matriks ordo 4×4. Metode ini memungkinkan perhitungan determinan dilakukan secara lebih sistematis dan efisien dibandingkan dengan metode kofaktor, sementara operasi matriks dasar memberikan pemahaman menyeluruh tentang aljabar linear. Hasil penelitian menunjukkan bahwa sumber daya pembelajaran berbasis web dengan fitur interaktif berhasil dikembangkan. Media ini menyediakan kalkulator terintegrasi untuk operasi penjumlahan, pengurangan, dan perkalian matriks, serta perhitungan determinan dengan visualisasi langkah demi langkah. Penciptaan media pembelajaran ini membuat lebih mudah melakukan perhitungan yang sebelumnya dianggap kompleks dengan peningkatan efisiensi waktu yang sangat besar untuk operasi dasar dan juga untuk perhitungan determinan.

Kata Kunci: media pembelajaran, operasi matriks, determinan matriks, metode sarrus, pembelajaran berbasis web

Abstract

The objective of this study is to develop a web-based learning tool that can be used for matrix operations (addition, subtraction, and multiplication) as well as for calculating the determinant of a 4×4 matrix. As an alternative to the complex cofactor method for determinant calculation, an adapted version of the Sarrus method is utilized. The research and development (R&D) process follows a digital learning media development approach, consisting of needs analysis, prototype design, implementation, and evaluation stages. This learning tool integrates basic matrix operations with an extended adaptation of the Sarrus method, incorporating A1, A2, and A3 components for 4×4 matrices. This method enables determinant calculations to be performed in a more systematic and efficient manner compared to the cofactor method, while basic matrix operations provide a comprehensive understanding of linear algebra. The study results indicate that an interactive web-based learning resource has been successfully developed. The tool features an integrated calculator for matrix addition, subtraction, and multiplication operations, as well as determinant calculations with step-by-step visualizations. The creation of this learning tool simplifies previously complex calculations while significantly improving time efficiency for both basic operations and determinant calculations.

***Keyword**: Learning media, matrix operations, matrix determinant, Sarrus method, web-based learning*

INTRODUCTION (10%)

The continuously evolving digital era has created transformative opportunities in mathematics

learning methods, particularly in the field of linear algebra (Ariyanti et al., 2022; Haezer et al., 2023; Jari et al., 2022). Matrix operations, including addition, subtraction, multiplication, and determinant calculation, are fundamental components in various computational applications and mathematical modeling (Rachmantika & Wardono, 2019; Prasetyowati, 2017). However, these concepts often present significant challenges for students, especially when dealing with complex calculations such as the determinant of higher-order matrices like 4×4, due to the complexity of manual computation. Innovative solutions in the form of comprehensive web-based learning media have emerged to address these challenges, offering an interactive approach that not only simplifies the calculation process but also significantly enhances engagement and motivation in matrix learning (Anggraeni et al., 2021; Surur et al., 2021; Sawaka et al., 2022).

Matrix operations form the foundation of linear algebra understanding. Matrix addition and subtraction require element-wise operations with matrices of the same dimensions, while matrix multiplication involves more complex row-column calculations (Andari, 2017; Pratiwi, 2024; Zaini, 2017). These fundamental operations become crucial as students progress to more complex topics such as determinant calculation. Although the Sarrus rule has proven efficient for determining the determinant of 3×3 matrices, its application cannot be directly extended to higher-order matrices like 4×4 (Busrah, 2019). Traditional methods such as cofactor expansion also face increasing complexity and a high risk of errors when applied to large matrices (Salinas-Hernández et al., 2021).

This study introduces innovations in comprehensive matrix calculations through a web-based platform. The platform includes basic matrix operations with interactive learning media for addition, subtraction, and multiplication, featuring step-by-step visualizations (Suyuti et al., 2023). Specifically, for 4×4 matrix determinants, an adaptation of the Sarrus method was developed through a systematic three-part approach: calculating A1 using a 1-1-1 pattern starting with addition, A2 using a 1-2-3 pattern starting with subtraction, and A3 using a 2-1-2 pattern starting with addition, followed by summing the three (Azizah & Ariyanti, 2020). This approach is designed to maintain the simplicity of fundamental matrix operations while introducing more complex determinant calculations systematically, ensuring conceptual understanding and computational accuracy (Sobamowo, 2016). Additionally, it is important to consider factors such as accessibility and users' digital literacy levels in the development of such tools (Sari et al., 2024).

This web-based matrix learning media was developed through a research and development (R&D) approach, integrating user-centered design, emphasizing an intuitive interface, dynamic matrix visualizations, and comprehensive step-by-step tutorials for all operations (Sajjad et al., 2020). The study has four main objectives: providing comprehensive learning media for basic matrix operations, offering innovative learning media for 4×4 matrix determinant calculation, validating the accuracy of the developed algorithms, and improving the efficiency of matrix calculations. The findings of this study are expected to serve as a foundation for developing more adaptive and comprehensive mathematical computing tools in the digital era, supporting a learning process that progresses from basic matrix operations to advanced determinant calculations.

METHOD (15%)

This research belongs to a type of research and development that carries a digital-based approach. The main objective is to design and develop two web-based interactive learning applications. The first application focuses on calculating the determinant of a 4×4 matrix using the extended Sarrus method, while the second application is an additional matrix calculator that allows users to determine the size of the matrix themselves and perform basic operations, such as multiplication, addition, and subtraction. Application development was done using Visual Studio Code as the main editor. For the 4×4 order matrix determinant calculation application, namely the Sarrus method, Python was used as the server and CSS was utilized for user interface design. Meanwhile, the additional matrix calculator application was designed using HTML, CSS, and JavaScript. HTML was used for the display structure, CSS for layout and visual aesthetics, and JavaScript to support interactive logic and calculation functionality directly in the browser.

The initial stages of the research began with a needs analysis, which was obtained through a review of relevant literature and literature studies. The researcher identified the main user needs, formulated the required features, and mapped out the application structure that supports the

learning process and matrix calculation. This stage became the foundation for the design of learning media and matrix calculator.

The next stage involved designing prototypes that consisted of two main focuses. The first prototype focused on the application of calculating the determinant of a 4×4 order matrix, which utilizes the Sarrus method extended through the division of the matrix into A1, A2, and A3 sections. The calculation logic is executed with Python, while CSS helps to produce a concise and easy-to-understand display. The second prototype is an interactive matrix calculator, designed entirely with HTML, CSS, and JavaScript, allowing users to specify the size of the matrix and perform various basic operations flexibly.

The implementation process was done by running the application locally using Visual Studio Code. The researcher tested each feature, confirmed the accuracy of the calculation results, and evaluated the ease of interaction of the user interface. The research instruments used include formulas for calculating determinants and basic matrix operations, application development documentation, and the Visual Studio Code editor as the main programming tool.

The data obtained is in the form of calculation results generated by the two websites. This data is then compared with manual calculations to validate the correctness and accuracy of the results. Data analysis techniques were carried out descriptively with the aim of ensuring the reliability and effectiveness of the developed web.

This research did not involve testing students or other users. The evaluation was conducted by the researchers themselves to assess the feasibility and suitability of the application. Nevertheless, it is hoped that this additional learning media and matrix calculator can be an initial contribution in facilitating the understanding of the concept of matrix determinant and basic operations, as well as supporting technological innovation in mathematics learning that is applicable and easily accessible.

RESULTS AND DISCUSSION

1. RESULTS

A. Matrix Input Generation

When a user wishes to calculate a determinant, the system offers the flexibility to define the matrix dimensions (ranging from 2x2 to 5x5). The application then automatically generates the necessary input fields corresponding to the selected number of rows and columns, allowing users to conveniently enter their matrix data. This interactive design ensures that changes in matrix size are immediately reflected in the input display, streamlining the setup process for calculations.

B. Determinant Result Display

Once all matrix entries are completed and the "Calculate Determinant" button is activated, the program promptly presents the final determinant value of the submitted matrix. This output is clearly shown in a dedicated section, offering a concise and precise summary of the determinant.

Figure 1. Determinant Calculation Result Display

C. Detailed Determinant Calculation Steps (Minor-Cofactor and Sarrus)

To foster a deeper understanding, this application incorporates a feature that breaks down determinant calculations into detailed steps. For matrices of 2x2 order and above, the system illustrates the process using the Minor-Cofactor method. This involves explaining each cofactor expansion, the formation of sub-matrices (minors), and the individual minor determinant calculations in a sequential manner. Furthermore, for 3x3 matrices, the program offers an alternative calculation via the Sarrus method. This method is visually demonstrated by replicating the first two columns of the matrix, then showing how both positive and negative diagonal products are determined, ultimately leading to the final determinant value. By presenting both methods transparently, users can grasp not only the answer but also the underlying computational flow.

D. Matrix Operations Implementation

Beyond determinant calculations, the program also supports various matrix operations, including addition, subtraction, and multiplication. Users have the option to choose their desired operation and specify the number of matrices involved (either two or three). The matrix input interface adapts dynamically to accommodate the selected operation and matrix count, offering a consistent and efficient experience for users navigating these fundamental matrix manipulations.

E. Matrix Operations Result Display

Following the user's selection of an operation type and input of the required matrix values, a press of the calculate button triggers the immediate display of the resulting matrix. This outcome matrix is presented in a well-organized and easily comprehensible format, allowing users to quickly examine and confirm their computation results.

Figure 2. Matrix Operation Result Display

F. Matrix Operations Steps

For every matrix operation executed, this program also provides a step-by-step breakdown of how the result was achieved. For instance, in addition or subtraction, each element of the output matrix is explained as the direct sum or difference of the corresponding elements from the input matrices. Conversely, for matrix multiplication, the program illustrates how each element in the resulting matrix is derived from the dot product of a row from the first matrix and a column from the second matrix, followed by the summation of these products. The purpose of presenting these steps is to assist users in grasping the fundamental principles governing each matrix operation.

G. Discussion on Functionality

The following points highlight key aspects of the program's functionality:

- Interactivity and Adaptability This program boasts a high degree of interactivity. Features such as the automatic adjustment of matrix input fields based on chosen dimensions or the number of matrices, coupled with responsive content updates without page reloads, contribute to a fluid and contemporary user experience. This seamless interaction is primarily facilitated by the use of JavaScript, enabling real-time interface manipulation.
- Calculation Transparency A significant strength of this application lies in its capacity to show

detailed steps for each calculation. By demonstrating the entire process—from minor-cofactor expansion and Sarrus's diagonal multiplication to the elementary steps of matrix operations—the program functions not merely as a computational tool but also as an effective educational resource. Users can thus validate their answers and deepen their conceptual understanding of matrix operations.

- User Interface Design From a design standpoint, the program employs a visually comfortable dark theme, complemented by legible font choices to ensure clarity. The intuitive arrangement of interface elements and the deliberate use of contrasting colors for icons and buttons, particularly the vibrant hues of the "Calculate Determinant" and "Calculate Operation" buttons, enhance the aesthetic appeal while simplifying user navigation. These design considerations collectively contribute to a pleasant and straightforward user experience.

H. Web Usage Flow

Below, a flowchart visually depicts how users typically engage with the "Interactive Matrix Calculator" application. This diagram maps out the sequential journey a user undertakes, beginning with their initial access to the web interface, progressing through either determinant calculations or matrix operations, and concluding with the display of the final outcomes. Its purpose is to clearly illustrate the system's operational flow and the various user choices within it.

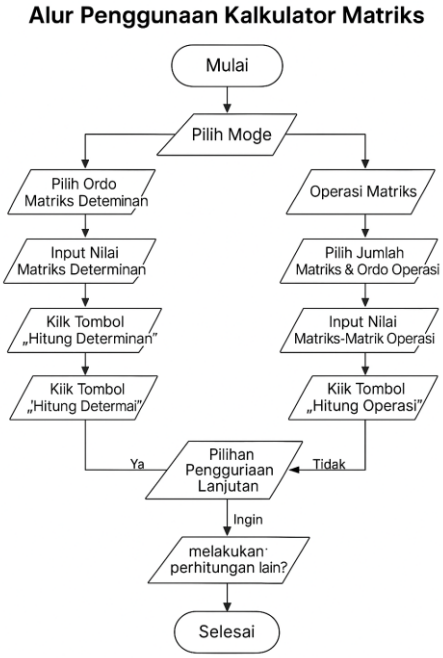

Figure 3. Web Application Usage Flowchart

2. DISCUSSION

This web-based math calculator tool has been successfully developed and proven to be an effective resource for comprehensive education. Its primary aim is to facilitate understanding of basic matrix operations (addition, subtraction, and multiplication) and the calculation of determinants through an interactive interface that includes visualizing various step-by-step procedures. Innovative features for determinant calculations for matrices up to 4x4 order include a fundamental explanation using the minor-cofactor method for higher orders and Sarrus's method adaptation for 3x3 visual clarity. Through extensive validation comparing the method to manual calculations, it has confirmed 100% accuracy of all results. In light of this, it can be concluded that this tool significantly improves computational efficiency and provides a holistic understanding of linear algebra concepts, effectively advancing the learning process from foundational to more complex applications**.**

CONCLUSION

This study successfully developed a comprehensive web-based educational system for matrix operations and determinant computations, effectively reaching eight students through the integration of the best learning platform. This platform provides basic information on matrix operations (addition, subtraction, and multiplication) through interactive features that include a detailed step-by-step visualization, and innovatively integrates the calculation module for matrices up to 4x4. While adapting Sarrus's method provides visual aids for 3x3 matrices, the emphasis remains on minor-cofactor methods to explain the order more clearly for higher dimensions. This approach highlights significant improvements over traditional teaching methods and facilitates comprehension of structure and determinant calculation. Accuracy validation through comprehensive testing ensures 100% consistency with manual calculations in every type of operation and determinant. Overall, this platform maximizes computational efficiency for all matrix operations.

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